ED5260: Mechanics and Control of Robot Manipulators

Project Submission

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May 3, 2017

1 Problem Statement

Consider a 2RP manipulator which is used in spot welding and other applications in industries. These manipulators travel to several points in space carrying out certain job at each point. The motion is preferred to be optimal based on multiple factors. Some of the prominent factors are time optimality and energy optimality.

2 Introduction

Optimal control theory is used to generate optimal control laws which minimizes the cost acquired during a process. In this report we focus on optimal control of a planar 2R manipulator. Given two initial points in the configuration space, the problem is to generate an optimal trajectory in terms of control which minimizes (or maximizes) the cost function of the manipulator from the initial to the final state with constraints on torque. The costs might take different forms like total control energy or time taken or any other quantifiable parameter which is to be optimized.

3 Approaches

This problem of optimal control can be solved in broadly two methods i.e. direct methods and indirect methods. The direct method involves discretizing all the states and controls into finite number of parts and using the numerical ODE solvers like Runge Kutta or Euler to propagate the states in time.

The problem can also be solved using indirect methods from optimal control. In the heart of this method lies Pontryagin maximum principle, which lays the set of necessary conditions that gives a set of control actions that takes the manipulator to the specified point minimizing the cost. These methods are elaborated later in the report.

$$\boxed{J = \Phi\left[\,\mathbf{x}(t_0), t_0, \mathbf{x}(t_f), t_f\,\right] + \int_{t_0}^{t_f} \mathcal{L}\left[\,\mathbf{x}(t), \mathbf{u}(t), t\,\right] \mathrm{d}\,t}$$

Figure 1: General cost function of an optimal control problem

In most of the cases the differential equation governing the system are nonlinear and complicated, hence there is no analytical expression possible. Hence numerical methods become very important in solving optimal control problem.

The numerical methods used for solving optimal control can be broadly classified into two different categories

3.1 Direct Methods

In this method the entire problem is discretized and made into huge non linear program and this is solved using any optimiser. Normally a linear function is used to discretise the control and a cubic function is used to interpolate the states between the collocation points. This method is simple but leads in approximate solutions. These are typically solved as initial value problems and hence guarantees convergence in most of the cases.

3.2 Indirect methods

Indirect methods uses Pontryagin principle to solve the problem. The results of indirect methods are more accurate compared to direct methods. However this results in solving two point boundary value problem which becomes very complicated and leads to numerical issues.

3.3 Shooting method

Solving two point boundary value problems is a general setting for solving an optimal control problem using indirect method. This method involves guessing initial condition for the states, using this guess the states are propagated forward in time. The final states obtained for this particular guess is found. A error function is formulated. In order to bring this to zero Newton-Raphson method is used which iteratively improves the guess. Although seems simple the convergence is very poor if the final time is large. This can be forther divided into simple shooting and multiple shooting.

4 Model Predictive Control

The control action derived using theory of optimal control is generally open loop in nature,i.e there is no feedback. Such control action are not preferred since they are not robust in nature. In order to make it more robust, a nonlinear MPC algorithm is coded.

Model predictive control is an control algorithm wherein the control actions are calculated based only on the present state value and nothing else. The basic algorithm is that it predicts the state over certain horizon and manipulates the control values such that the error obtained is minimised. Once we get a control value, this is used to propagate the system forward in time and this is repeated again and again.

Results:

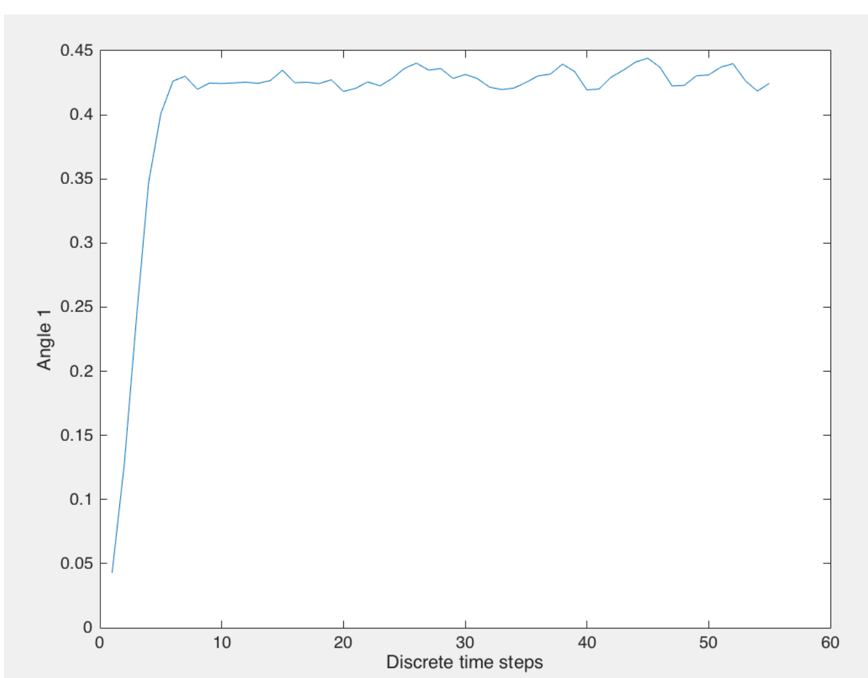

Figure 2: Robustness of MPC

5 Time optimal control of a 2R Manipulator

Using Pontryagins maximum principle, it can be verified that the control has to be bang - bang in nature because the Hamiltonian is affine with respect to the controls. This gives rise to another major issue of singularity with respect to control, which is explained in detail below

5.1 Singular optimal control

The control actions are decided based on a function called switching function which can be derived using the maximum principle.One major downfall of this method is it fails when the switching function is identically zero over finite interval of time. More complicated algorithms have been developed to take care of this case.

5.2 Time optimal study

The manipulator is required to move from (0,0) to (pi/5,0) in the configuration space. The corresponding time optimal trajectory generated is as shown.
State

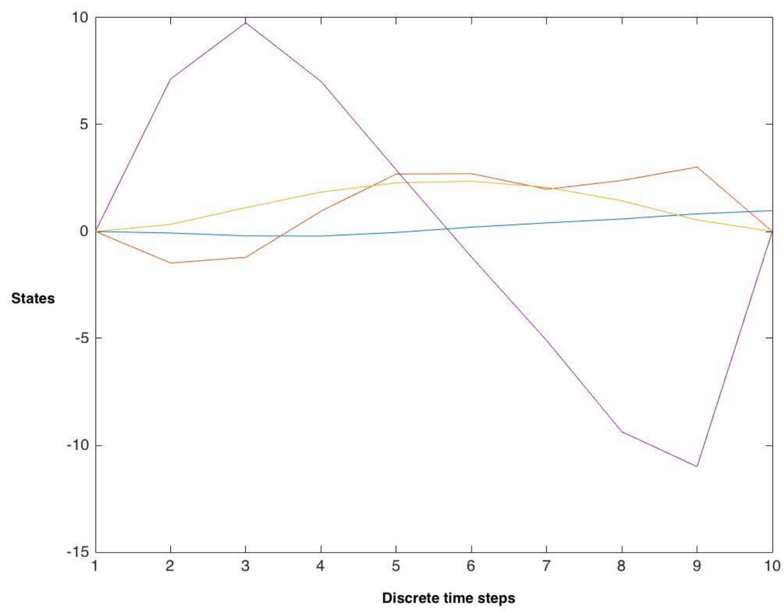

Figure 3: States

Trajectory:
The stroboscopic image of the motion of the manipulator is as shown in the figure Fig.4.

5.3 Validation of time optimal control

An indirect method is developed to find the optimal control law for a planar 2R manipulator. As discussed already this leads us to solve two point boundary value problem. Since the system is very complicated single shooting methods fails to converge, hence the initial guess for the costates were taken from [1]. Based on this guess the initial value problem is solved using Runge Kutta fourth order method. In order to verify the solution the Hamiltonian was plotted along the entire time.

6 Energy Optimal Control

A code has been written to find the energy optimal law of control by using direct method. For numerical propagation in time a simple Euler method has been used. Using this the states and the control in the next time instances are obtained and is continued till the final state is reached. Throughout the process, it is ensured that the constraints on states and control have been met. The results are plotted as a time history plot of a 2R manipulator.

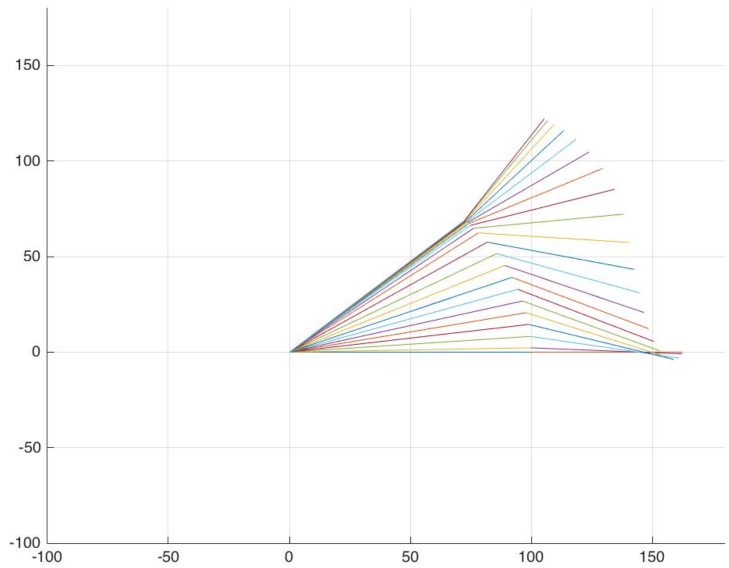

Figure 4: Time history of the manipulator

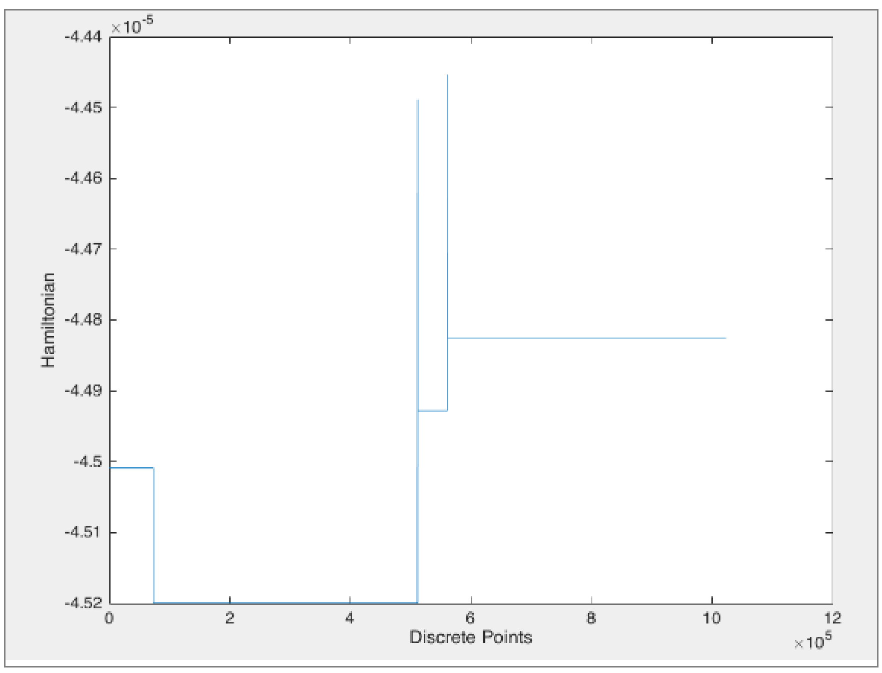

Figure 5: Hamiltonian of the resultant time optimal control law

6.1 Validation

The energy is plotted with time and is seen that it is almost constant. The variation is mainly due to the unknown dynamics used in the paper. Also the inbuilt MATLAB function fmincon was used for obtaining the optimal value during each iteration. The optimization has converged and has produced a local minima. Which says that the algorithm is valid.

Also the dynamics of the 2R manipulator used to find the energy has been validated for the Hamiltonian (Energy) and is found out to be close to zero throughout the range.

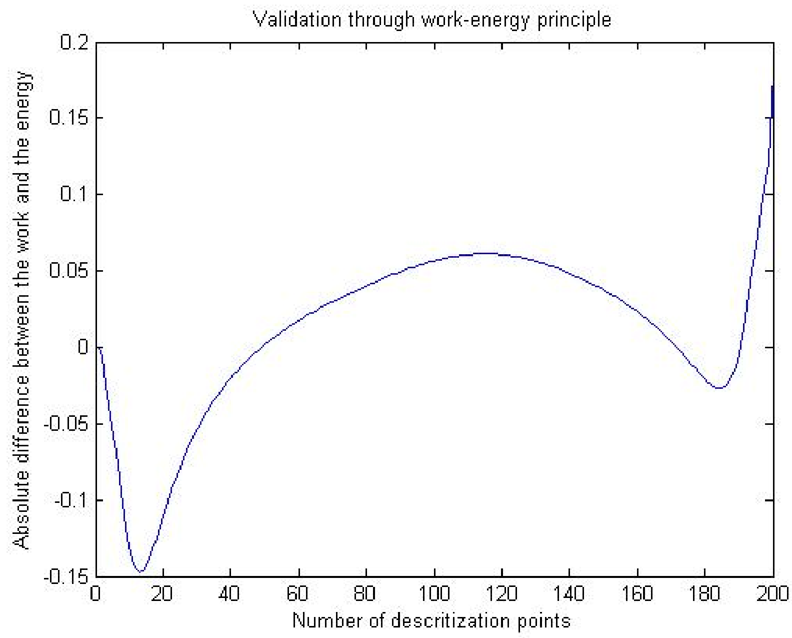

Figure 6: Validation through work energy principle

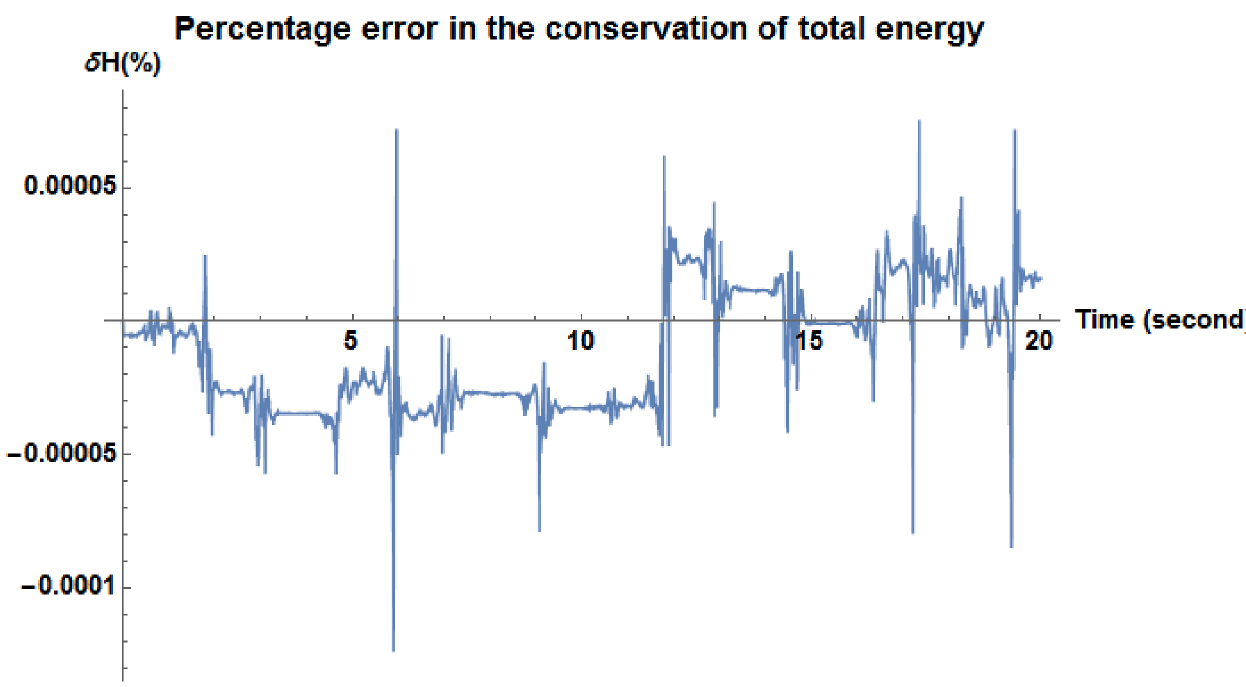

Figure 7: Energy validation of the dynamics equation derived

7 Study of energy optimal trajectory as mass of links change

It is taken that the mass of the whole setup to be 9 kg and the ratio of mass distributed as different in each case and the control and states are observed.
Case 1
mass of link 1 = 4.5kg
mass of link 2 = 4.5kg
Cost incurred in terms of control energy is : 27.9075
States:

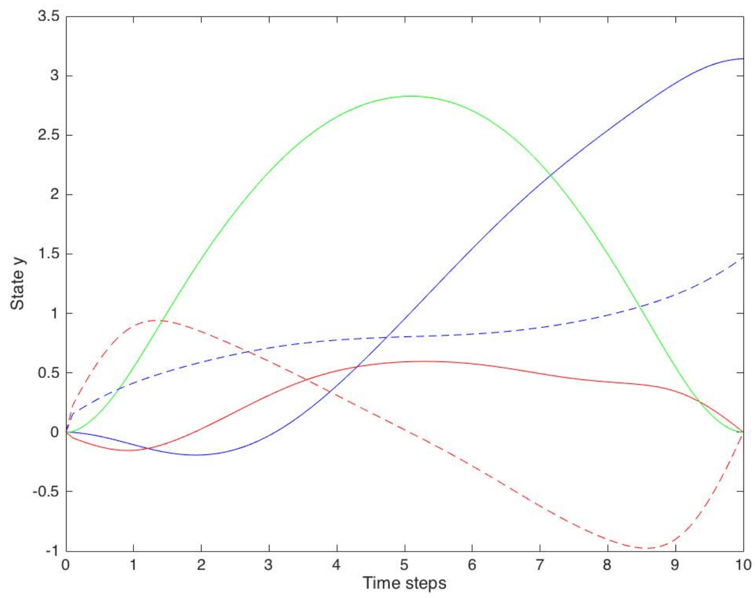

Figure 8: States for case 1

Controls:

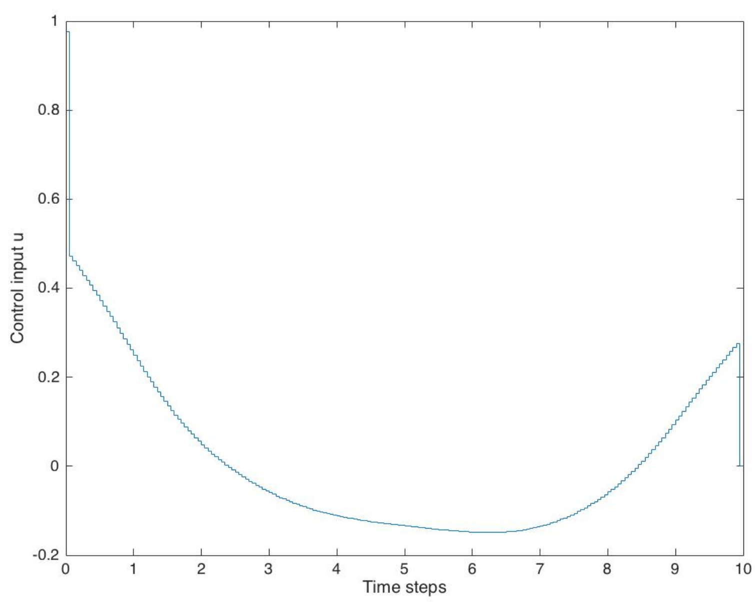

Figure 9: Control at link 1

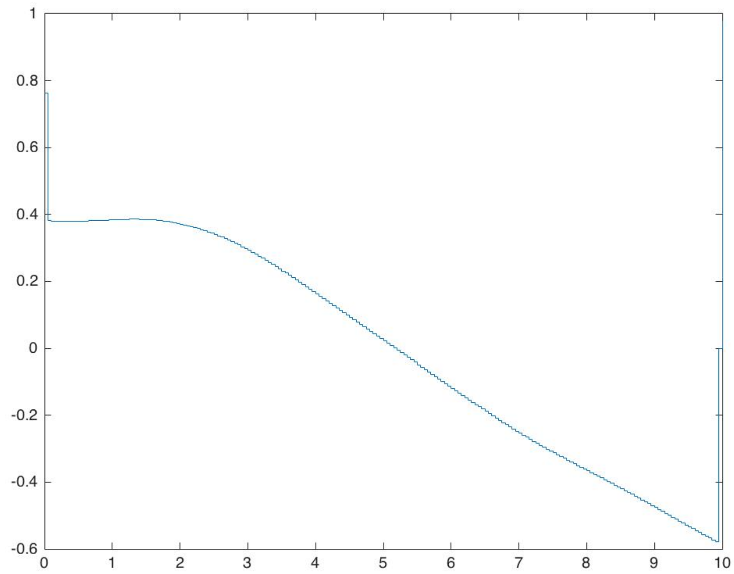

Figure 10: Control at link 2

Trajectory:

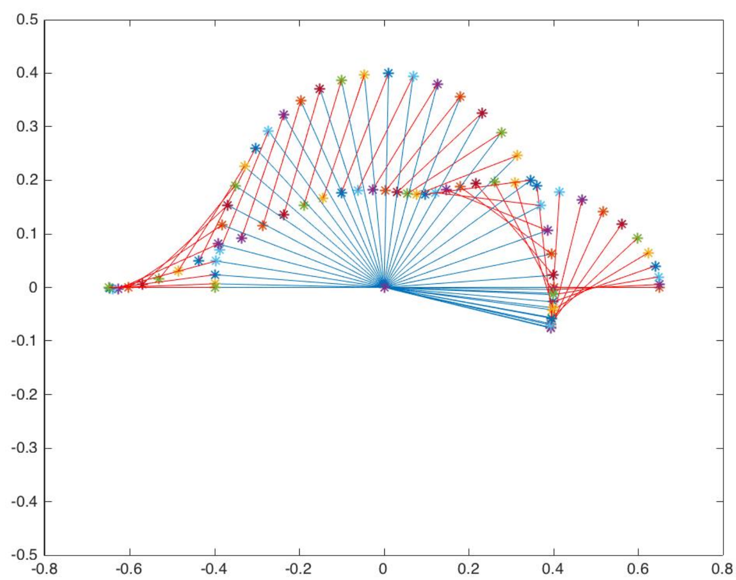

Figure 11: Time history plot of the manipulator

Case-2
mass of link 1 = 1kg
mass of link 2 = 8kg
Cost incurred in terms of control energy is : 29.8362
States:

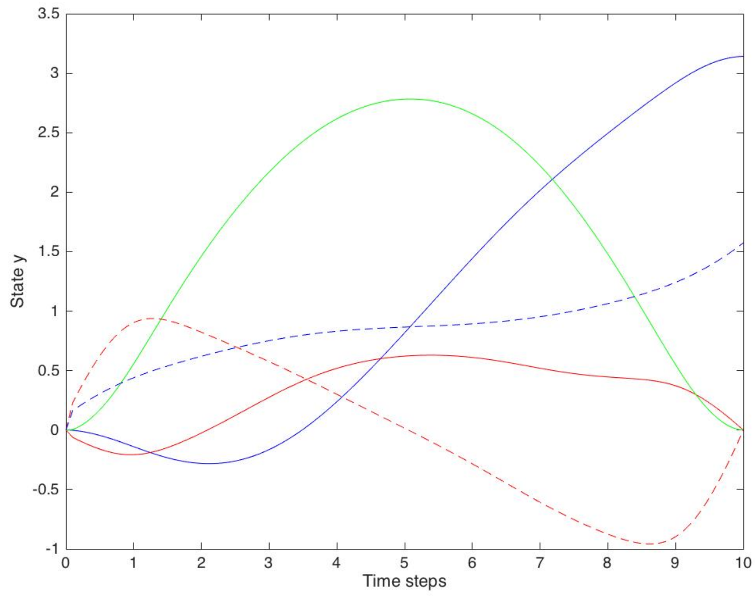

Figure 12: States for case 2

Controls:

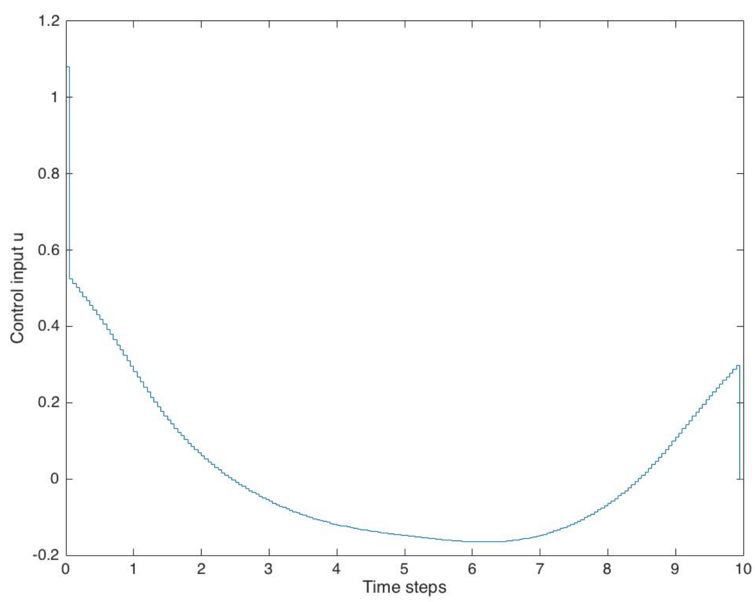

Figure 13: Controls at link 1

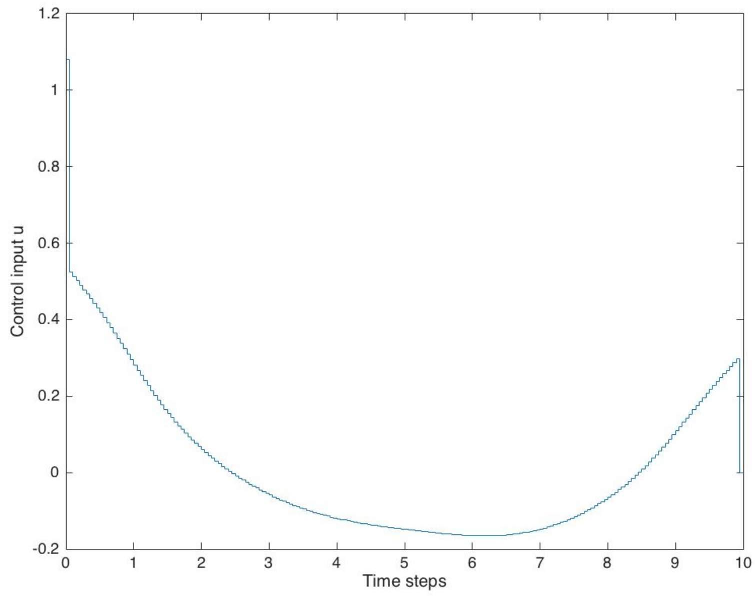

Figure 14: Control at link 2

Trajectory:

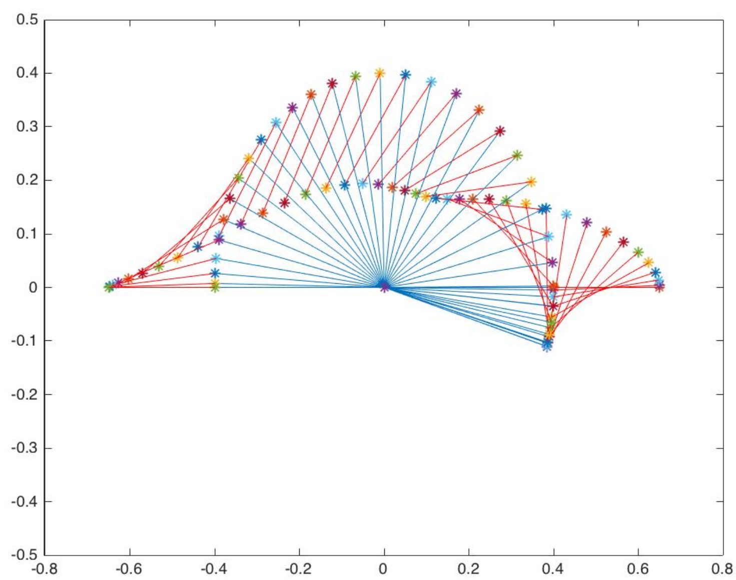

Figure 15: Time history of case 2

Case-3
mass of link 1 = 8kg
mass of link 2 = 1kg
Cost incurred in terms of control energy is : 25.3733
States:

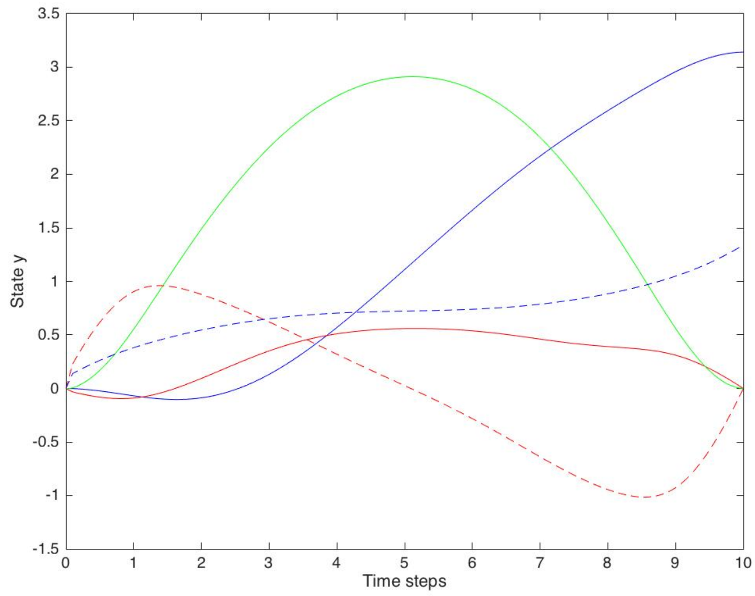

Figure 16: States for case 3

Controls:

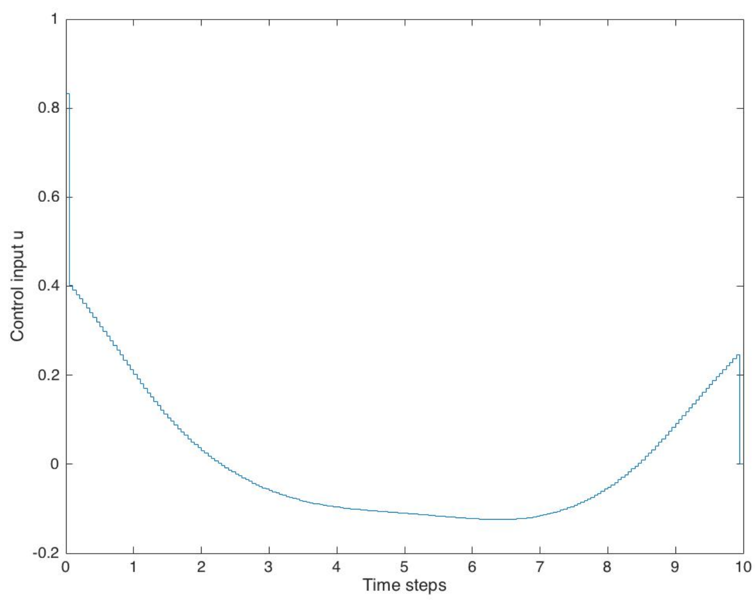

Figure 17: Control at link 1

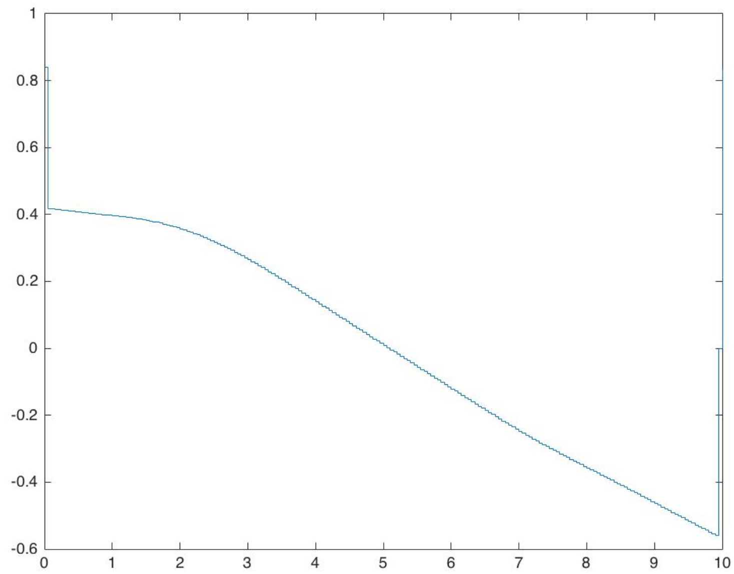

Figure 18: Control at link 2

Trajectory:

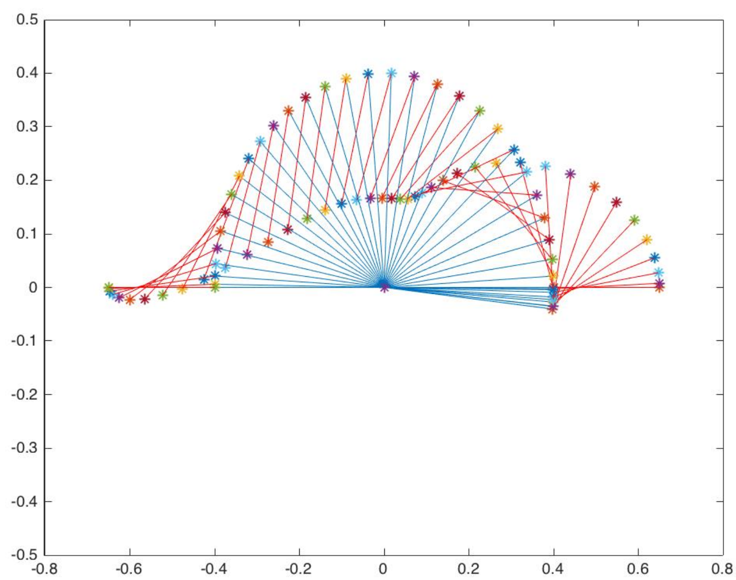

Figure 19: Time history for case 3

7.1 Observation

It is observed that as the mass of the link 1 is increase having the total mass of the system as constant, the cost keeps coming down as shown in Fig.18.

7.2 Inference

The observation seems reasonable because in case 1 most of the enrgy is located at the end or at the link 2 which means that the moment of inertia due to it is very high and high energy would be required at both the joints. But as the mass moves into link 1, moment of inertia reduces and hence the cost incurred.

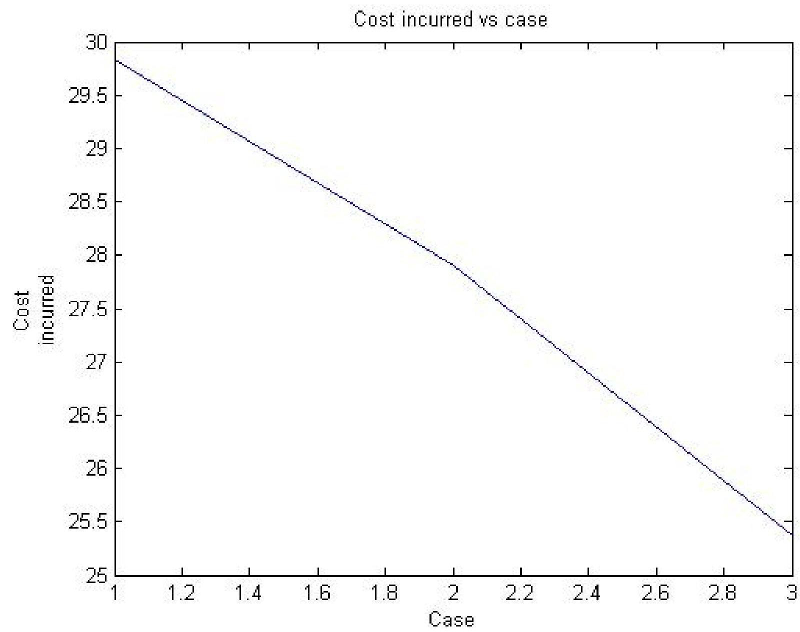

Figure 20: Cost incurred vs mass of link 1

8 Analysis of energy optimal control with change in link lengths

The analysis is done considering the total footprint of the manipulator to be a constant of 0.65m.

Case-1
Link length 1 = 0.1
Link length 2 = 0.55
Cost incurred in terms of control energy is : 46.1057
States:

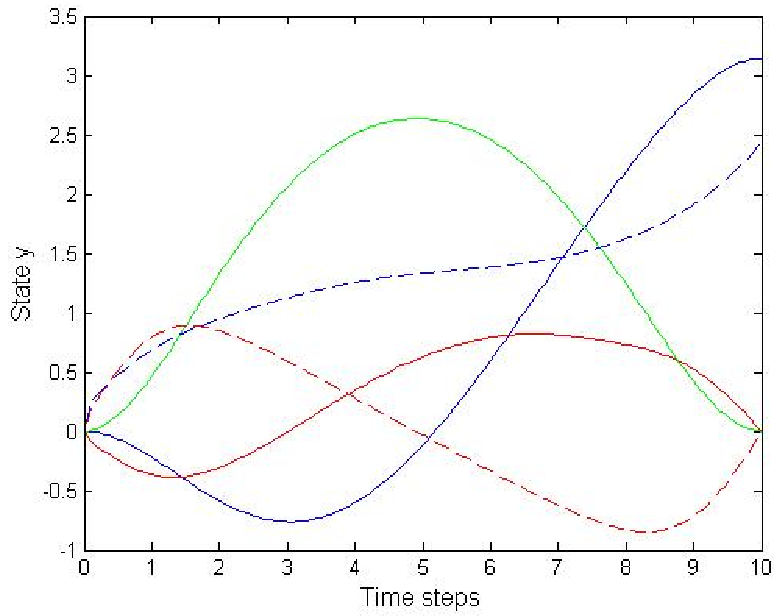

Figure 21: States

Controls:

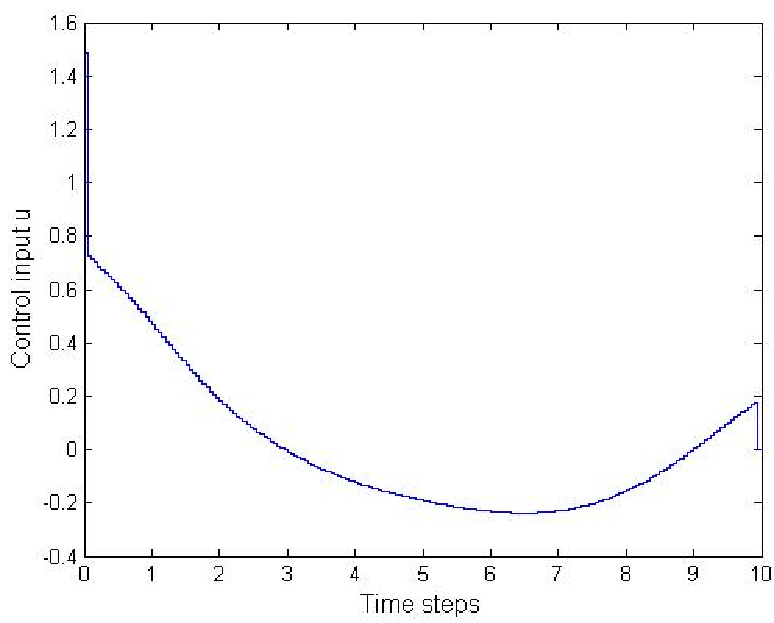

Figure 22: Control at link 1

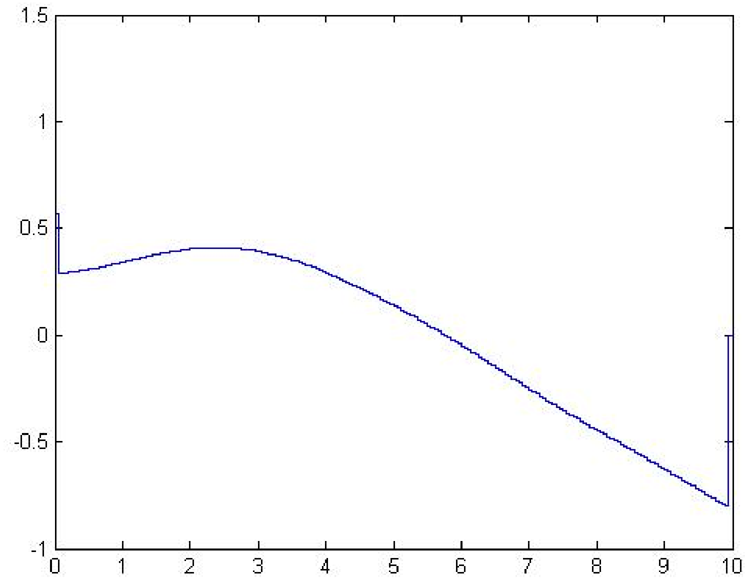

Figure 23: Control at link 2

Trajectory:

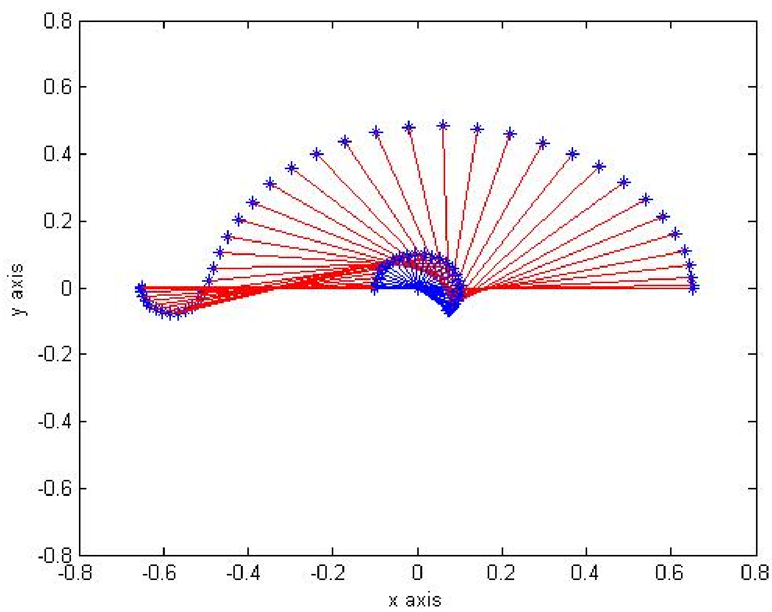

Figure 24: Time history plot

Case-2
Link length 1 = 0.35
Link length 2 = 0.3
Cost incurred in terms of control energy is : 31.173

States:

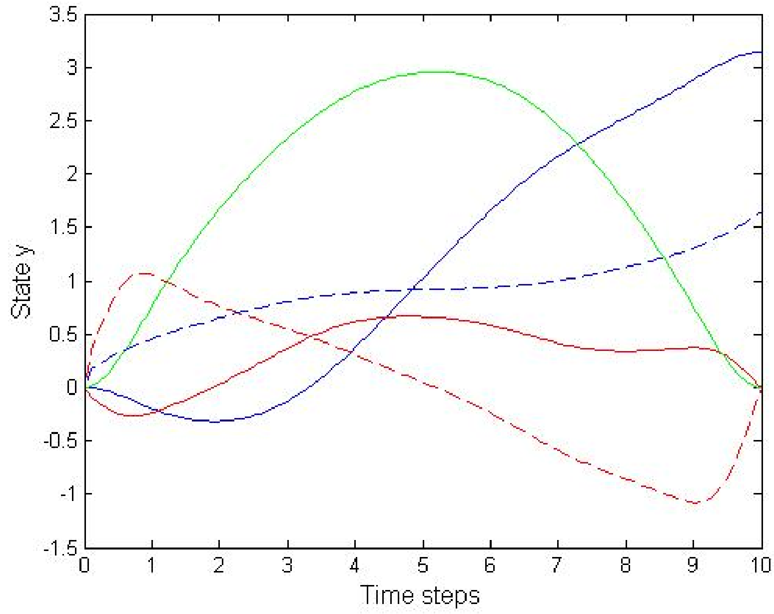

Figure 25: States

Controls:

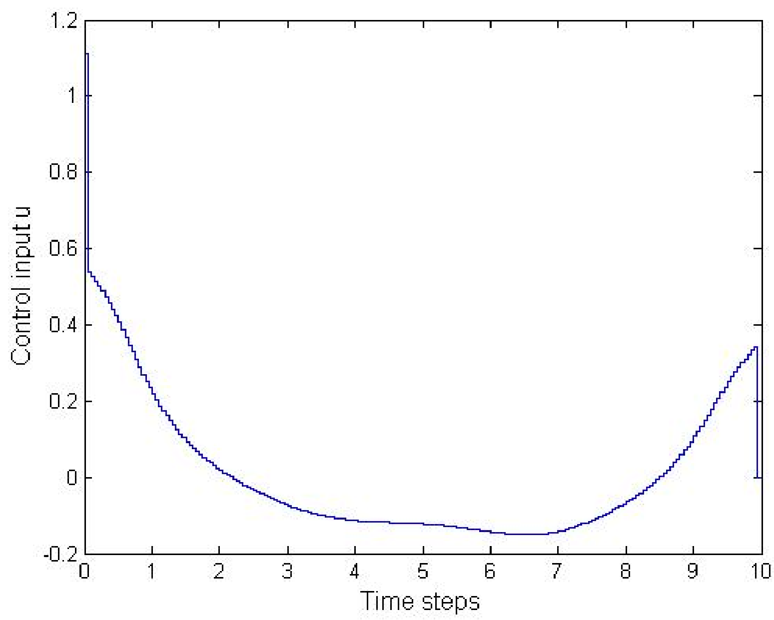

Figure 26: Control at link 1

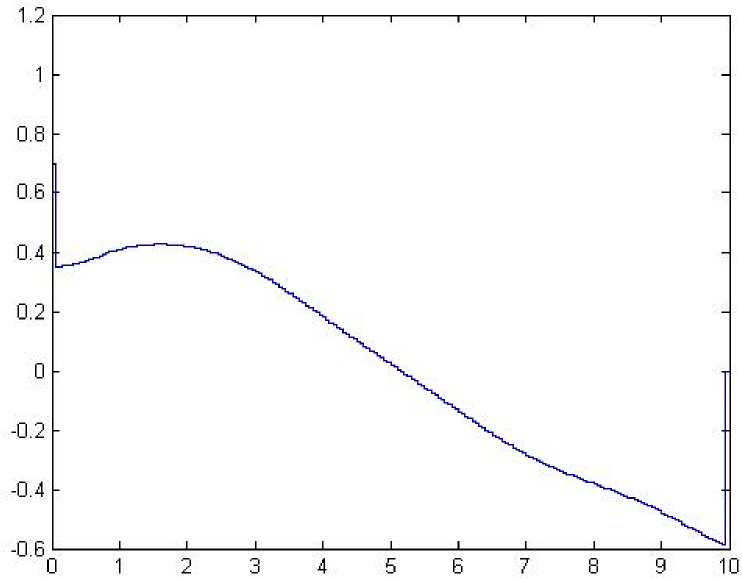

Figure 27: Control at link 2

Trajectory:

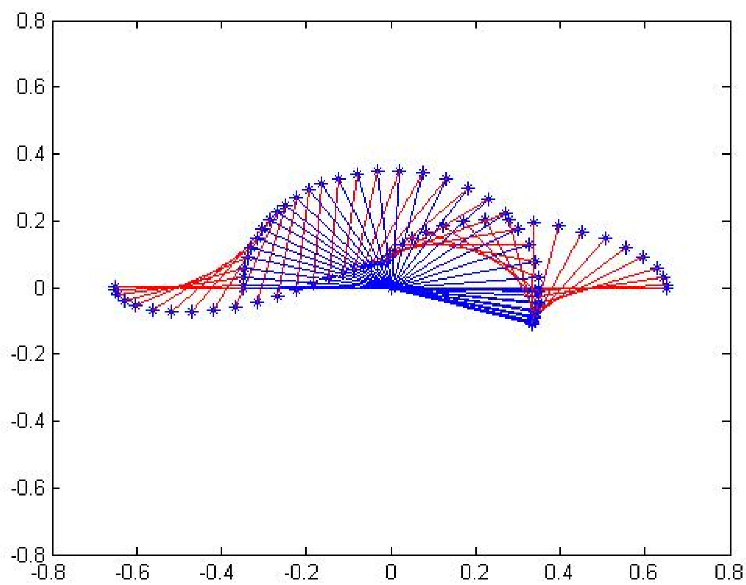

Figure 28: Trajectory for case 2

Case-3
Link length 1 = 0.55
Link length 2 = 0.1
Cost incurred in terms of control energy is : 139.958
States:

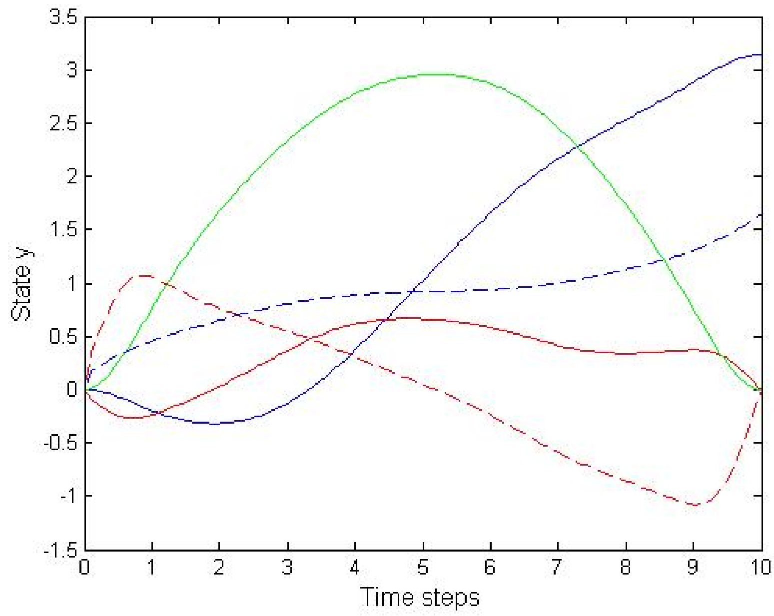

Figure 29: States

Controls:

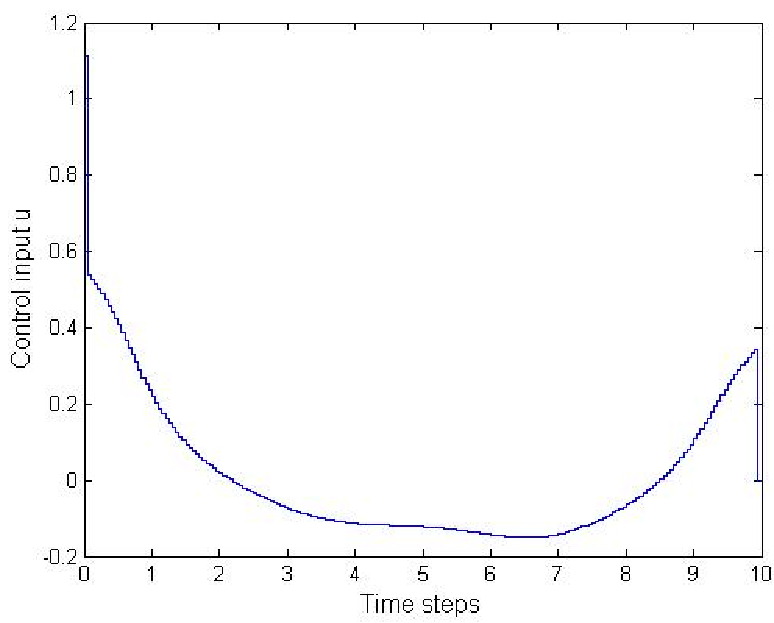

Figure 30: Control at link 1

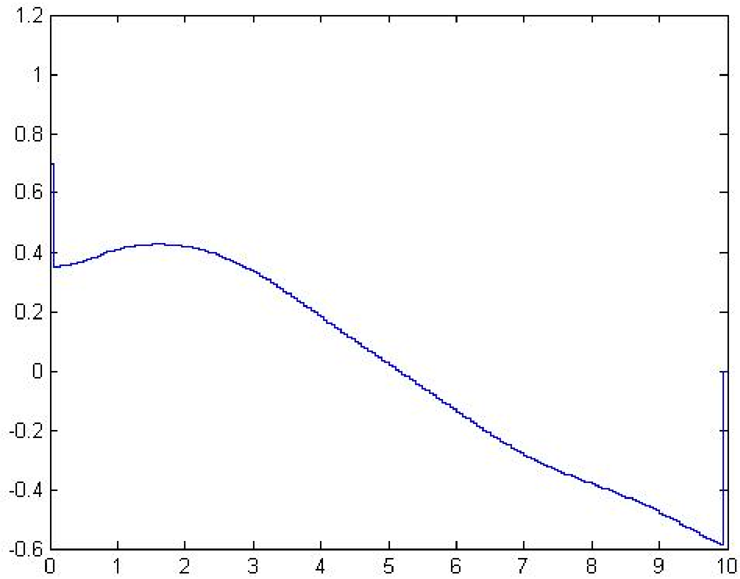

Figure 31: Control at link 2

Trajectory:

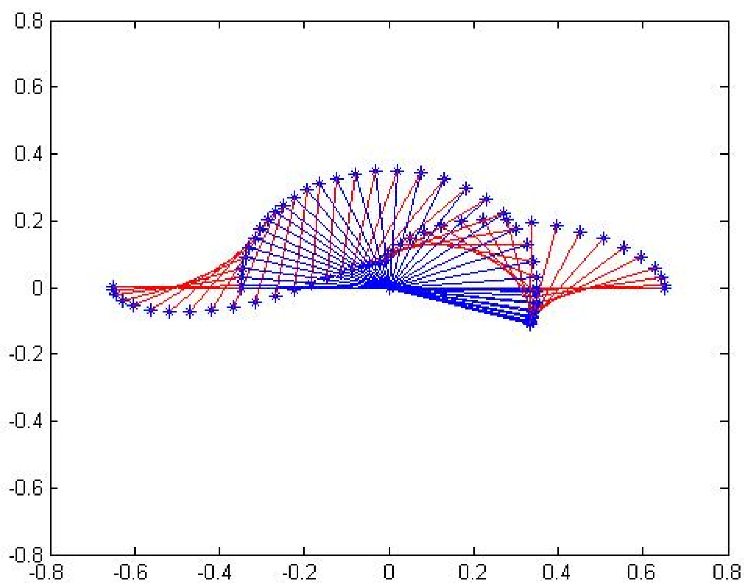

Figure 32: Trajectory for case 3

8.1 Observation

It can be observed from the cost incurred with change in length that as the length of link 1 increases the const decreases and then as it crosses some number it is seen that it increases again. The trend is as shown.

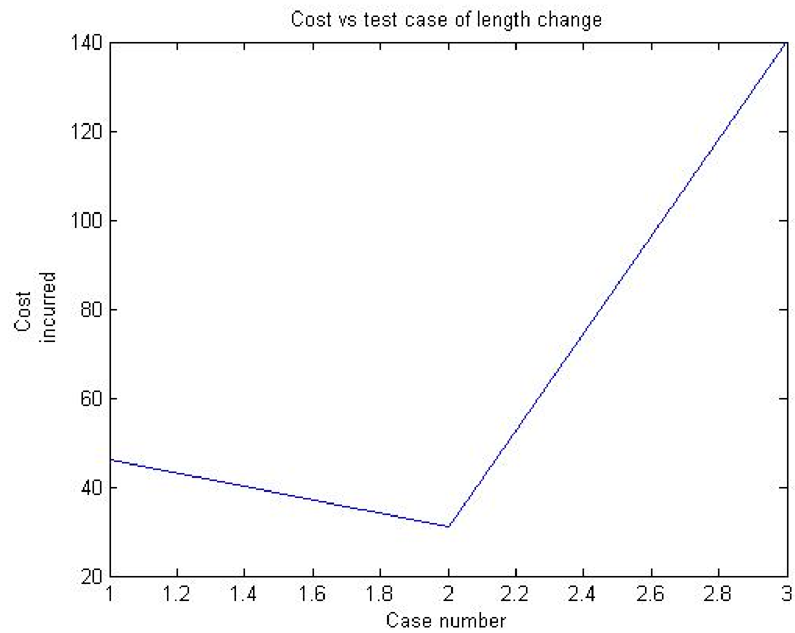

Figure 33: Change in cost as link lengths are varied

8.2 Inference

As the link length 1 is increased the control cost increases as its moment of inertia changes about the pivoted z axis. It is also observed that the cost incurred when l1 is very less is also higher than an intermediate value. This is due to higher energy required at the second motorowing to its increase in moment of inertia. But the rise of energy is not the same as the link masses are not identical.

9 References

1. H.P.Geering et. al.,"Time-Optimal Motions of Robots in Assembly", IEEE Transactions on automatic control, vol.31, no.6, June, 1986.
2. Bulirsh, "Introduction to Numerical Analysis", 3rd Edition.